Further notes on the Position of the Sun's Axis of Rotation, as deduced from observations by C. H. F. Peters in the years 1860–1870. Papers of the I.U.S.R. Computing Bureau. No. IV. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The observations made in 1860–1870 by Professor C. H. F. Peters, at Clinton, N.Y., with a 13-inch refractor, were edited by Professor E. B. Frost, and published in 1907 by the Carnegie Institution. The positions of the spots are given, but not their areas. The spots are identified from day to day by marginal letters. In the discussion which follows, all spots are included which persisted for at least 5 days, as indicated by the marginal letter. The total movement between the extreme days was entered, and the number of days; and these totals and numbers of days were all added up for each calendar month, grouping the spots in latitude as indicated in Table I. This table, therefore, does not correspond in detail to those given in the two previous papers for Greenwich measures (*Monthly Notices*, lxviii. p. 98 and p. 609), since the material is given in a different form, but the results will presently be compared with those previously obtained.

TABLE I.

Daily drift in Latitude in each month, during the period 1860–70.

(The unit is 0°·001.)

Latitude.	−25°	−20°	−15°	−10°	0°	+10°	+15°	+20°	+25°
Jan.	—	−117	− 62	− 36	− 18	−54	−41	+ 9	−29	—
Feb.	− 8	−179	+140	+ 79	+119	− 6	+25	+20	+55	+ 11
Mar.	+125	− 56	+ 75	− 36	+ 49	+37	+43	+59	− 4	− 43
Apr.	− 48	+ 13	+ 29	− 51	+ 42	+12	+18	+71	+54	− 19
May	0	+ 10	− 96	+ 41	+ 35	+59	+28	−60	+59	+ 62
June	+ 33	+ 38	+ 1	+ 46	+ 71	+ 4	− 8	−17	+69	− 20
July	—	+ 14	+ 37	+121	+ 33	− 9	− 7	+14	+62	− 12
Aug.	+ 44	− 20	+ 33	− 17	+ 14	−42	+23	+32	+74	+ 89
Sept.	− 38	+ 86	− 49	+ 7	+ 43	−45	+71	−18	+29	− 100
Oct.	+ 66	+ 60	− 6	+ 8	− 21	−38	−39	+41	+93	+111
Nov.	+183	− 75	−128	+ 45	− 18	+56	−76	−29	+13	—
Dec.	—	−125	− 50	− 20	− 84	−40	− 1	+56	−80	—
Means			− 6	+ 16	+ 22	− 6	+ 3	+15		

2. Excluding the outside groups (for latitudes above 20°) we can now analyse the remaining six columns harmonically, to find an expression of the form

$$A \sin \theta + B \cos \theta + C,$$

where $\theta = 0$ for the middle of January.

The results for A and B are as in Table II., those for C being given at the foot of the columns.

<div align="center">TABLE II.</div>

<div align="center">*Values of the first harmonic in units of $0°.001$.*</div>

Latitude.	$-20°$	$-15°$	$-10°$	$0°$	$+10°$	$+15°$	$+20°$	Mean.
A . . .	$+41$	-3	$+41$	$+27$	$+20$	$+4$		$+22$
B . . .	-1	-25	-20	-2	-15	$+17$		-8

3. We must now consider what errors in the position of the Sun's axis are hereby indicated. Suppose that in the middle of any particular month the distance of the central meridian of the Sun's disc from the node where the adopted equator cuts the real equator is θ, and that the inclination of the two equators is a'. Then the separation of the equators on the central meridian is $a' \sin \theta$. In longitude β before the central meridian it is $a' \sin (\theta - \beta)$, and in longitude β after the central meridian it is $a' \sin (\theta + \beta)$, and the apparent drift of a spot between these longitudes is thus

$$a' \left\{ \sin (\theta + \beta) - \sin (\theta - \beta) \right\} = 2a' \sin \beta \cos \theta.$$

If we group together all such drifts for a whole month, the value of θ will change by $\pi/6$, say from $\theta - \pi/12$ to $\theta + \pi/12$; and the average value of the drift is got by integrating $\cos \theta \, d\theta$ between these limits and dividing by the interval. This is easily seen to introduce the factor

$$R = \frac{12}{\pi} \sin \frac{\pi}{12} = 1 - \cdot 012,$$

which has hitherto been neglected as sufficiently near to unity. It can easily be restored if need be.

4. In previous papers β was definitely taken to be either $65°$ or $35°$, and the results have been sufficiently accordant. Hence, in the present instance, where the material is scantier, different values of β have been grouped together to make the most of the observations. Spots which live on the Sun for 5 days or more have all been grouped together, and the average value of β is the angle turned through by the Sun in about 4 days, *i.e.* about $53°\cdot3$. The precise value does not matter very much, since β only enters through the factor $\beta/\sin \beta$, owing to the fact that we have divided by the total number of days when we should have used $2 \sin \beta$. The value of this factor at $50°$ is $1\cdot13$, and at $60°$ is $1\cdot21$. Probably $1\cdot15$ is sufficiently exact for our present purpose, and the reciprocal is $0\cdot87$. Taking the angle of rotation in one day as $13°\cdot3$, an error of a' in the adopted position of the axis will thus cause a periodic error in the drift for one day, as measured in the present investigation, of

$$a' \times 0\cdot87 \sin 13°\cdot3 = a' \times \cdot20.$$

We thus have to multiply A and B by ·060 to convert them into minutes, and then divide by 0·20 to compare with *a*, the resultant factor being ·30. Hence the mean A is $+6''·6$, and the mean B $-2'·4$.

5. We can now compare the various results as follows, adding Carrington's as starting-point.

Mean Errors of Adopted Position of Sun's Axis.

		A.	B.
Carrington	1853–1861	$0'·0$	$0'·0$
Peters	1860–1870	$+6·6$	$-2·4$
Greenwich	1874–1885	$+7·3$	$-0·6$
Greenwich	1886–1901	$+7·4$	$-7·8$
[Greenwich	1902–1905	$+12·4$	$-1·7$]

The results for 1902–1905 have not yet been published in detail, and are quite provisional. The reduction was made in order to see whether they would support the large negative value of B given by the period 1886–1901. They apparently do not: an independent reduction including all spots (instead of only those between $+20°$ and $-20°$) gave a positive value for B, viz. $A = +7'·0$, $B = +7'·2$. The value of B is, as has already been remarked, much more uncertain than that of A, as it depends on the comparison of summer and winter photographs.

6. If we consider that the observations as a whole indicate an error in Carrington's adopted axis represented by

$$A = +7'·0 \qquad B = -3'·6,$$

then the errors of the determinations from different zones may be tabulated as follows:—

Errors of Determinations.

	A.			B.		
	$-20°$ to	$-10°$ to	$+10°$ to $+20°$	$-20°$ to	$-10°$ to	$+10°$ to $+20°$
1860–1870	$-1'·3$	$+3'·2$	$-3'·4$	$-0'·3$	$+0'·3$	$+3'·9$
1874–1885	$-6·9$	$+1·0$	$+6·8$	$+10·8$	$+3·3$	$-5·6$
1886–1901	$-3·8$	$-1·2$	$+6·3$	$-3·7$	$-9·9$	$-5·1$

The sums of the squares of the nine residuals for A is 174 and for B is 312, which shows the greater uncertainty in the determination of B.

7. One of the important points presenting itself for consideration before a definitive solution can be undertaken is the question of the discordance between the values of A and B from spots north and south of the equator. Up to the present the material has been discussed in a provisional manner in order to see where any systematic differences lay, so that such groups might be kept separate in the final discussion. The grouping of the observations is a matter of great importance in dealing with sun-spots, for near minimum the material is scanty and confined to special latitudes, so that systematic differences may seriously affect the general mean.

Now there seems to be a curious systematic difference between spots north and south of the equator. Let us take the element A, which is better determined than B. We have for the above three periods—

1860–1870	$A_N - A_S = -2'\!.1$
1874–1885	$A_N - A_S = +13'\!.7$
1886–1901	$A_N - A_S = +10'\!.1$

$$\text{Mean} \quad +7'\!.2$$

The first period is discordant, so that the difference might be an idiosyncrasy of the Greenwich results. To test this point solutions were made from the spots of over 20° in latitude, which were excluded from the above solutions, and thus give an entirely independent check. The results were

	From spots over ± 20°.	Previous results (double weight).	Mean.
1860–70	$A_N - A_S = +20'\!.4$	$-2'\!.1$	$+5'\!.4$
1874–85	$A_N - A_S = +17'\!.5$	$+13'\!.7$	$+15'\!.0$
1886–01	$A_N - A_S = +4'\!.2$	$+10'\!.1$	$+8'\!.1$
Means	$+14'\!.0$	$+7'\!.2$	$+9'\!.5$

Hence these spots in high latitudes indicate that the small value of $A_N - A_S$ previously found from the period 1860–1870 is accidental; and that on the whole there is a real discordance between spots N and S of the equator. We must thoroughly investigate this discordance, for until the origin of it is understood it is impossible to assign any definite meaning to the deduced position of the Sun's axis, since we are liable to get a different position according as the spots preponderate in one hemisphere or the other. The next step accordingly will be to go over the material again with this point specially in mind. See also *M.N.*, lxviii. p. 101.

8. Coming now to the constant C, the mean value from the six columns in Table I. between latitudes is $+7'\!.3$, which means a drift of $0°\!.0073$ per day, or $+0°\!.20 = +12'$ per rotation. Thus the tentative hypothesis put forward (in *Mon. Not.*, lxviii. p. 103) of a 26 years oscillation must be given up.

There has apparently been only one change of sign in this drift since Carrington's time, and the figures in the paper quoted should be expanded as follows :—

Mean Drift during a whole Rotation.

Carrington.	Peters.	Greenwich.					
1853–1861.	1860–1870.	1874–9.	1880–5.	1886–9.	1890–3.	1894–7.	1898–01.
$+18'$	$+12'$	$+12'$	$+36'$	$-30'$	$-32'$	$-56'$	$-30'$

A special examination must therefore be made of the results near 1885, when the change occurred.

Second Note on the number of Faint Stars with large Proper Motions. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. Last November the results of a comparison of ten pairs of plates in zone $+28°$ taken at intervals of 12 to 15 years were communicated to the Society. Out of 2822 stars examined, 16 showed P.Ms. greater than $15''$ per century, and 6 greater than $20''$ per century. Of the 16 stars, 7 were brighter than $9·5$ magnitude; and of the 6 with P.M. $>20''$, 3 were bright.

2. The comparison and examination of 30 plates in zone $+26°$ has now been carried out, and I am again glad to acknowledge the volunteer assistance of Mr. J. H. Worthington, F.R.A.S., and Mr. G. H. Hamilton, in remeasuring and checking the cases of possible proper motion. Table I., which follows, shows the general nature of the results. As a rule, all cases have been specially examined where the differences between the plates exceeded $\pm 1''·2$ in either coordinate, and the figures in column 4 give the number of cases where these differences were confirmed on remeasurement and thus probably indicate a P.M., although no stress is laid on these records unless the P.M. exceeds $15''$ per century.

TABLE I.

Zone $+26°$. Stars having probable large P.Ms.

R.A. of Plate Centre.		Total Stars compared.	Interval.	Total P.Ms.	Centennial P.Ms.	
h	m				$15''-20''$.	$>20''$.
0	8	162	12·1	3	3	0
0	24	199	14·0	0	0	0
1	4	183	10·9	0	0	0
1	12	227	10·1	1	0	1
1	52	132	8·8	1	0	1
5	36	349	13·1	6	0	0
6	48	303	11·0	5	2	1
7	12	191	9·9	2	1	1
7	44	159	13·0	4	2	0
8	0	145	14·8	5	1	1
8	16	127	15·0	8	2	2
9	4	98	14·9	4	1	3
9	28	126	14·9	2	1	0
10	8	84	12·0	1	0	1
10	24	83	12·0	1	1	0
11	12	65	14·9	5	2	0
12	0	72	14·9	4	0	0
13	4	54	14·9	3	1	0
13	12	35	14·9	5	0	1